First meeting of Project IODISSEE

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Etucational History

- Undergraduate Study
  - 2002-2006 at Wuhan University, China
  - Major in Mathematics and Applied Mathematics
    Minor in Computer Science and Technology
- Graduate Study
  - 2006-2008 at Université de sciences et technologies de Lille, France
  - Major in Applied Mathematics
  - Master's Thesis concerning Image Enhancement and Noise Removal Problems under the guidance of Christophe BESSE
- PhD Study
  - 2008- at Université de sciences et technologies de Lille, France
  - Subject: Models and Numerical schemes for effect of the ionospheric perturbations in earth-satellite communications under the guidance of Christophe BESSE

We consider a two dimensional elliptic equation presenting large anisotropies:

$$\begin{cases} -\nabla \cdot (\mathbb{A} \nabla \phi) = f, \\ \frac{\partial \phi}{\partial z} = 0 \text{ on } \partial \Omega_z, \\ \phi = 0 \text{ on } \partial \Omega_x. \end{cases}$$

with

$$\mathbb{A} = \begin{pmatrix} A_\perp & 0 \\ 0 & \frac{1}{\varepsilon} A_z \end{pmatrix}$$

where $A_\perp$ and A_z are functions of the (x, z) coordinates with comparable order of magnitudes.

Summary of my work from September 2008 to December 2009

The Asymptotic Preserving Scheme

- First Task: Constant $\varepsilon \ll 1$
  - Passing $\varepsilon \to 0$ leads a ill-posed problem
  - Decomposing ϕ to $\bar{\phi} + \phi'$ gives a mean equation, a fluctuation equation and a constraint $\overline{\phi'} = 0$ called Asymptotic preserving Scheme which is well-posed independently on ε
- Second Task: Functional ε
  In the pratical case, the parameter ε depends on the coordinates, thus we should change slightly our AP scheme

A commonly used model in ionospheric plasma modeling is the 'Dynamo' model which can be written in 2 parts:

3-dimensional Transport Equation

$$\frac{\partial w}{\partial t} + \nabla \cdot (vw) = 0,$$

$$v_\alpha = \frac{u_{i\alpha}}{r \sin \varphi}, \; v_\beta = r \sin \varphi |B| u_{i\beta}, \; v_\gamma = |B| u_{i\gamma}$$

$$\text{where } w = \frac{\rho}{|B|^2}$$

Summary of my work from September 2008 to December 2009

The Dynamo Model in Ionosphere

3-dimensional Elliptic Equation

$$-\nabla \cdot (\mathbb{A} \nabla \phi) = -\nabla \cdot J_n,$$

where ϕ is the electric potential,

$$\mathbb{A} = \begin{pmatrix} \mathcal{A} & -\mathcal{D} & 0 \\ \mathcal{D} & \mathcal{B} & 0 \\ 0 & 0 & \mathcal{C} \end{pmatrix}$$

$$\mathcal{A} = \frac{\rho(\mu_i^P + \mu_e^P)}{r^2 \sin^2 \varphi |B|^2}, \ \mathcal{B} = \rho(\mu_i^P + \mu_e^P) r^2 \sin^2 \varphi,$$

$$\mathcal{C} = \rho(\mu_e^{\|} + \mu_i^{\|}), \ \mathcal{D} = \rho(\mu_i^H - \mu_e^H)/|B|.$$

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The Dynamo Model in Ionosphere

Here, the 3-dimensional elliptic equation is an anisotopic problem since $\frac{\mathcal{A}}{\mathcal{C}}$ and $\frac{\mathcal{B}}{\mathcal{C}}$ vary from 10^{-2} to 10^{-14}. Thus we try the two following methods

- Searching for the limit model: Striation Model
  - easy to archieve
  - work only in ambient ionosphere
- Applying the AP scheme to 3-dimensional Dynamo model